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A GRAPHOANALYTICAL METHOD FOR CALCULATING THE PROCESS OF SOLIDIFICATION OF STEEL INGOTS

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A graphoanalytical method to calculate the process of solidification of an ingot is developed. The solution is performed for the problem formulated in a unified statement. As a mathematical apparatus, the method of equivalent sources is used.

At present, among the wide range of nonlinear problems for determining the temperature field in a multiphase system involving moving interfaces where liberation (absorption) of heat occurs, one must note two competent mathematical formulations of the problem: the Stefan statement (with a smooth interface) and the problem of heat conduction with phase transitions in a temperature range.

By virtue of the nonlinearity (the presence of discontinuities in the fields of heat fluxes at the interface; the dependence of the law of movement of the phase-transition front and the thermophysical properties on unknown functions), the Stefan problem, in principle, cannot have exact analytical solutions. At the same time, the solution of the Stefan problem makes it possible to determine the duration of complete solidification of an alloy, the thickness and the temperature of a solidified layer, the velocity of movement of the crystallization front, etc.

The statement of the problem of heat conduction with phase transitions in a temperature range takes into account the realization of the phase transition not along a line but in a layer limited by the range of temperatures $T_{sol} \le T \le T_{liq}$. Thus, a calculational procedure of this type for the process of ingot solidification enables one to predict the appearance of structural defects of a metal which arise precisely in the above layer. A numerical solution of the problem of heat conduction with phase transitions enables one to carry out investigations of the simulated process of solidification in multidimensional bodies of a complex geometric shape.

Despite the large calculation error in solving applied problems of metallurgical production, one has to resort rather frequently to an approximate solution of the problems precisely in the Stefan statement. As a rule, one proceeds in a similar way when it is necessary to use the solution of the corresponding heat-conduction problem as a starting point for studying any other problem (for example, calculation of temperature stresses).

There are many approximate methods intended for solving the Stefan problem: the L. S. Leibenzon first method [1] (it is based on the assumption of the quasistationary state of the temperature field in a solid phase), the S. M. Adams method of successive approximations [2] (the exact solution of the classical self-similar Stefan problem is used), the A. I. Veinik method of elimination of variables [3] (it presupposes a priori assignment of the temperature-distribution function in a solidified layer), the B. Ya. Lyubov method [4] (based on the series expansion of the temperature function of a solid phase in the difference of the coordinates of the of an arbitrary point and the crystallization front), the L. S. Leibenzon [5] and G. P. Ivantsov [6]

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second method (the L. S. Leibenzon first method is refined by means of an artificial increase in the latent heat of solidification to account for the influence of the physical heat of a solidified layer), the I. D. Semikin and É. M. Gol'dfarb correcting method [7] (the heat content of a liquid phase is taken into account), the method of an instantaneous regular regime, the M. Biot variational method [8], etc.

The above methods differ from each other by the degree of complexity and accuracy of the results. Therefore, the question of the development of new analytical approaches to solution of Stefan-type problems aimed at simplifying and increasing accuracy remains topical as before.

In the present work, we investigated thermal processes of crystallization of a massive ingot using the method of equivalent sources, which showed a good performance in solving a wide class of linear and nonlinear boundary-value problems of heat conduction [9]. The method pertains to a group of equivalent sources of approximate analytical methods for solution of the problem of heat conduction. In conformity with the principle of a thermal boundary layer, the process of heating (cooling) is divided into two stages: the inertia stage (during which heating (cooling) of a body occurs over the cross section) and the regular stage (during which the temperature of a thermomassive ingot increases (decreases) further).

In the inertia stage of heating (cooling) ($0 \le \mathrm{Fo} \le \mathrm{Fo}_0$), the temperature field over the body cross section has two zones:

on the unheated (uncooled) part of the ingot cross section ($0 \le x \le b(\mathrm{Fo})$): $\theta(x, \mathrm{Fo}) \cong \theta_0$;

on the heated (cooled) part of the ingot cross section ($b(\mathrm{Fo}) \le x \le 1$): $\theta(x, \mathrm{Fo}) = \theta_1(x, \mathrm{Fo})$.

In the regular stage ($\mathrm{Fo}_0 \le \mathrm{Fo} < \infty$), the temperature field is described by the single temperature function $\theta_2(x, \mathrm{Fo})$. Its initial value is determined from the condition $\theta_2(x, \mathrm{Fo}_0) = \theta_1(x, \mathrm{Fo}_0)$.

To determine the form of the functions $\theta_1(x, \mathrm{Fo})$ and $\theta_2(x, \mathrm{Fo})$, in each stage we write an equation of the type

$$\frac{1}{x^m}\frac{\partial}{\partial x}\left(x^m \frac{\partial \theta_i}{\partial x}\right) = f_i(\mathrm{Fo}), \quad i = 1, 2,$$

(where $f_i(\mathrm{Fo})$ are the equivalent sources), which is solved relative to θ_i. The method of equivalent sources is described in [10] in greater detail.

In comparison with other methods of a thermal layer, the method of equivalent sources has the following advantages: an explicit determination of an unknown function; accuracy of calculations suitable for practice (the error is 5–6%).

In developing the procedure for solution of the problem of metal solidification, we considered ingots of basic geometric shapes: a plate, a cylinder, and a sphere. However, such an approach is easily generalized to any massive bodies of arbitrary configuration. It will suffice to remember that many scientists and metallurgists (for example, see [3, 4, 7]) suggest replacing the ingot studied by a cylinder with the equivalent radius

$$R_{\mathrm{eq}} = \sqrt{F/\pi},$$

where F is the mean cross-sectional area of the body.

The solution of the problem of solidification of a cylindrical ingot (Fig. 1) using the method of equivalent sources has the following form [11]:

the duration of complete solidification of the ingot is

$$\bar{\tau} = \tau_{\mathrm{tot}} - \tau^0 = \tau^* (1 + \bar{\beta}_1 + \bar{\beta}_2), \tag{1}$$

the temperature-field distribution in the solidified layer of the ingot and on the surface is

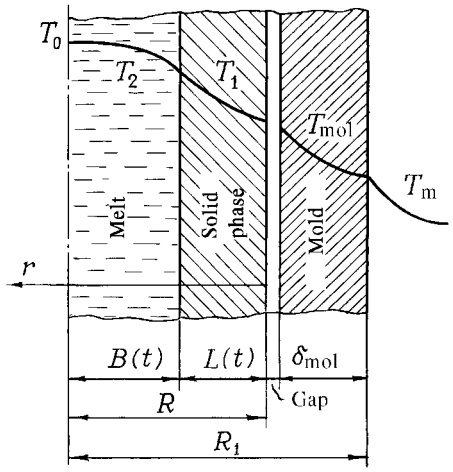


Fig. 1. Computational scheme of the process of solidification of an ingot.

$$\theta_1(\xi, \tau) = \theta_1^*(\xi, l)\left\{1 - \frac{V(l)\,[1 - \Phi(\xi, l)/\theta_1^*(\xi, l)]}{\mathrm{Ko}\,[1 + \omega_1(l) + \omega_2(l)]}\right\}, \tag{2}$$

$$\theta_{sur}(l) = \Phi_{sur}(l) + \frac{\Psi_{sur}(l)}{1 + \omega_1(l) + \omega_2(l)}, \tag{3}$$

where

$$V(l) = \frac{\mathrm{Ko}\,\omega_1(l)}{\{1 + m\,[1 - l(\tau)]\}\,l(\tau)}\;\frac{1 + \dfrac{\mathrm{Bi}_1\,l(\tau)\,[1 - l(\tau)]}{2\,[3 + \mathrm{Bi}_1\,l(\tau)]}}{1 + \dfrac{\mathrm{Bi}_1\,l(\tau)\,[1 - l(\tau)]}{2 + \mathrm{Bi}_1\,l(\tau)}};\quad \Phi(\xi, l) = \frac{2 + \mathrm{Bi}_1\,\xi\,(2 - \xi)}{2 + \mathrm{Bi}_1\,l(\tau)\,[2 - l(\tau)]};$$

the position of the crystallization front $l(\tau)$ at the instant of time τ is

$$H^*(l)\,[1 + \beta_1(l) + \beta_2(l)] = \frac{\tau - \tau^0}{K_a\,\mathrm{Ko}}, \tag{4}$$

the duration of cooling of a superheated melt to the instant of the beginning of solidification is

$$\tau^0 = \frac{1}{3\,(1 + m)\,\mathrm{Bi}_2}\left[\left(1 + \frac{\mathrm{Bi}_2}{4}\right) - \frac{2 + 3\,\mathrm{Bi}_2 + \mathrm{Bi}_2^2}{\mathrm{Bi}_2}\ln\left(1 + \frac{\mathrm{Bi}_2}{2}\right) + (3 + \mathrm{Bi}_2)\ln(1 + K_T)\right]. \tag{5}$$

The proposed solution takes into account the heat which is lost by the solid phase in the course of intrinsic cooling and the heat content of the liquid phase (superheating). It is obvious from formula (1) that the presence of such factors can substantially slow down the crystallization process.

In [11], we present formulas for calculating the functions and criteria that enter into Eqs. (1)–(4). However, for the convenience of practical implementation it seems worthwhile to express their components in graphic form as well, which considerably simplifies the calculational procedure. This approach makes it possible to perform calculations for any parameters Ko and Po (Po is Yu. S. Postol'nik's criterion) and to

carry out interpolation only by the Bi_1 criterion. In the case of determining the duration of complete crystallization of an ingot, it becomes unnecessary to perform this interpolation.

Practical experience shows that for massive steel ingots cooled in a mold and in air, the Bi_1 criterion does not exceed unity. Therefore, in constructing graphs we took Bi_1 = 0.25, 0.5, 0.75, and 1.0. The intermediate values can easily be obtained by means of the simplest linear interpolation.

As an example of applying the proposed procedure, we present a calculation of a specific ingot. The initial data are as follows: weight of the ingot 8 tons, steel grade St. 45, type of mold open-bottom cast iron, ingot dimensions $0.734 \times 0.655 \times 2.8$ m, thickness of the mold wall δ_{mol} = 37 mm, pouring temperature of the melt T_0 = 1530°C, liquidus temperature T_{liq} = 1490°C, solidus temperature T_{sol} = 1420°C, the temperature in the gap T_g = 900°C, heating temperature of the mold T^0_{mol} = 150°C, and temperature of the ambient air T_m = 90°C; the latent crystallization heat is taken to be equal to q = 270 kJ/kg.

Commercial experiments show that the temperature drop between the inner and outer surfaces of the mold depending on the ingot, the mold, pouring conditions, etc. can reach a considerable value (of the order of 300°C). The inner mold surface has a temperature of 700–800°C, while in the initial period and on the separate portions of the inner-mold surface it has a temperature close to the melting temperature of cast iron (1100°C). The outer-surface temperature is 400–500°C (in pouring into a cold mold) or exceeds this value by the heating temperature of the mold (in pouring into a hot mold). Therefore, the main function of the heating, i.e., decrease in the temperature drop, makes it possible to decrease the thermal stresses. In our case, we take ΔT_{mol} = 250°C.

Then

$$T^{out}_{mol} = (400 + 500)/2 + 150 = 600\ ^{o}\text{C}; \quad T^{in}_{mol} = T^{out}_{mol} + \Delta T_{mol} = 600 + 250 = 850\ ^{o}\text{C}.$$

We determine the rough mean temperature of the solidified metal layer as

$$\overline{T}_1 = (T_{liq} + T_g)/2 = (1490 + 900)/2 = 1195\ ^{o}\text{C}.$$

For the temperature of the steel $\overline{T}_1$ = 1195°C its thermophysical characteristics, i.e., thermal conductivity, heat capacity, and density, have the following values: $\overline{\lambda}_1$ = 29.8 W/(m·deg), $\overline{c}_1$ = 0.66 kJ/(kg·deg), and $\overline{\rho}_1$ = 7.41 tons/m^3, respectively.

Now we calculate the mean value of the thermal diffusivity for the solidified metal layer and the liquid phase (for $T_2 = T_0$):

$$\overline{a}_1 = \overline{\lambda}_1/(\overline{c}_1\,\overline{\rho}_1) = 29.8 \cdot 3600/(0.6610^3 \cdot 7.41 \cdot 10^3) = 0.0219\ \text{m}^2/\text{h};$$

$$\overline{a}_2 = \overline{\lambda}_2/(\overline{c}_2\,\overline{\rho}_2) = 30.6 \cdot 3600/(0.72 \cdot 10^3 \cdot 7 \cdot 10^3) = 0.0219\ \text{m}^2/\text{h};$$

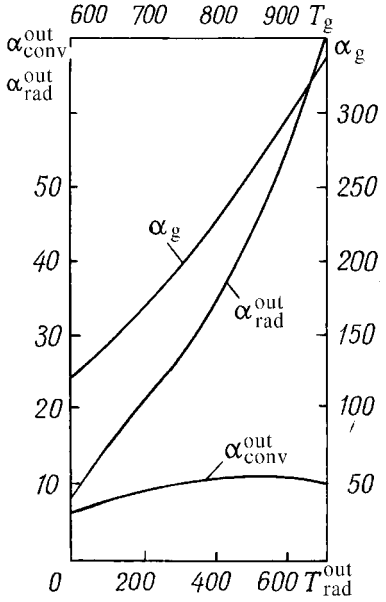
$$K_a = 0.0219/0.0219 = 1.$$

The reduced heat-transfer coefficient α_{1eq} is calculated from the formula [3]

$$\alpha_{1eq} = \left(\frac{1}{\alpha_g} + \frac{R_{eq}}{\lambda_{mol}} \ln\frac{R_{mol}}{R_{eq}} + \frac{R_{eq}}{R_{mol}\,(\alpha^{out}_{rad} + \alpha^{out}_{conv})}\right).$$

The unknown values are determined from the graphs (Fig. 2):

$$\alpha_g = 300\ \text{W}/(\text{m}^2\cdot\text{deg}); \quad \alpha^{out}_{rad} = 55\ \text{W}/(\text{m}^2\cdot\text{deg}); \quad \alpha^{out}_{conv} = 11\ \text{W}/(\text{m}^2\cdot\text{deg});$$



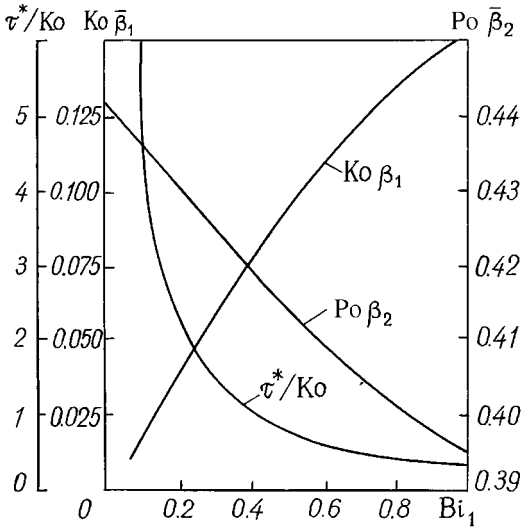


Fig. 2. Dependence of the heat-transfer coefficients α_g, α_{conv}^{out}, and α_{rad}^{out} on the temperatures. T, °C; α, W/(m^2·deg).

Fig. 3. Graphs of the change in τ^*/Ko, Ko $\bar{\beta}_1$, and Po $\bar{\beta}_2$ as a function of the criterion Bi_1.

$$\alpha_{1eq} = \left(\frac{1}{300} + \frac{0.391}{35.5} \ln \frac{0.761}{0.391} + \frac{0.391}{0.771\,(55+11)} \right) = 54.1 \ \text{W/(m}^2\text{·deg)} .$$

Then we evaluate the necessary criteria:

$$\text{Bi}_1 = \frac{\alpha_{1eq} R_{eq}}{\bar{\lambda}_1} = \frac{54.1 \cdot 0.391}{29.8} = 0.71 ;$$

$$\text{Ko} = \frac{q\bar{\rho}_2}{\bar{c}_1 \bar{\rho}_1 (T_{sol} - T_m)} = \frac{270 \cdot 7}{0.66 \cdot 7.41\,(1420 - 30)} = 0.28 ;$$

$$\text{Po} = \frac{q}{\bar{c}_2/(T_0 - T_{liq})} = \frac{270}{0.72\,(1530 - 1490)} = 9.4 .$$

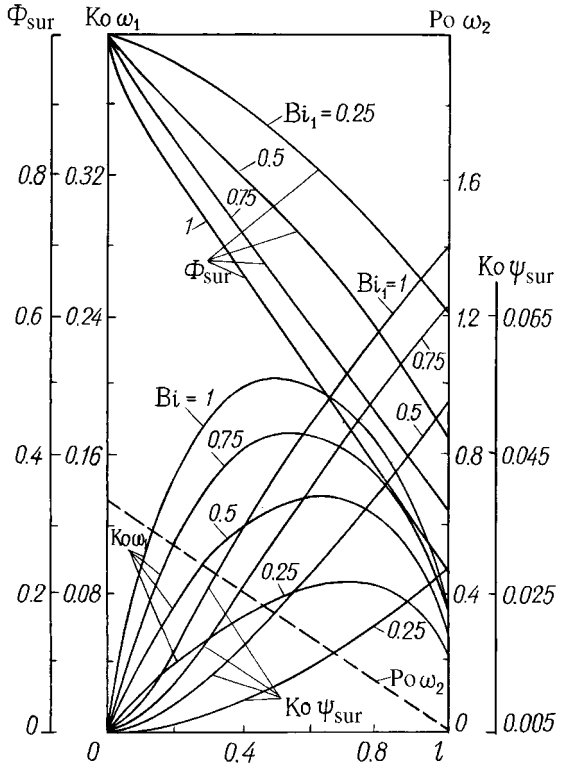
From the graphs given in Fig. 3 we determine that

$$H^* = \tau^*/\text{Ko} = 0.95 ; \quad \tau^* = 0.95 \cdot 0.28 = 0.266 ;$$

$$\bar{\beta}_1 \, \text{Ko} = 0.12 ; \quad \bar{\beta}_1 = 0.12/0.28 = 0.432 ;$$

$$\bar{\beta}_2 \, \text{Po} = 0.404 ; \quad \bar{\beta}_2 = 0.404/9.4 = 0.043 .$$

The time of complete solidification in relative $\bar{\tau}$ and absolute units $\bar{t}$ can be calculated from formula (1):



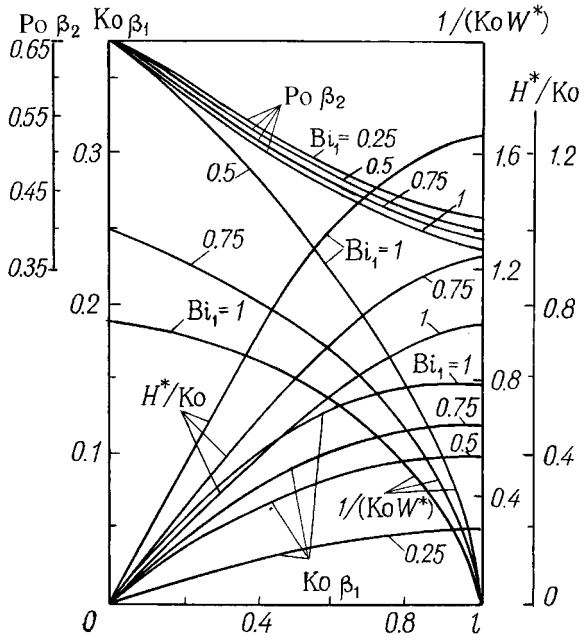


Fig. 4. Graphs of the change in Φ_{sur}, Ko ω_1, Po ω_2, and Ko Ψ_{sur} as a function of the solidified-layer thickness.

Fig. 5. Change in H^*/Ko, Ko β_1, Po β_2, and 1/(Ko W^*) as a function of the solidified-layer thickness.

$$\bar{\tau} = 0.266 \cdot (1 + 0.432 + 0.043) = 0.392 ; \quad \bar{t} = \frac{\bar{\tau} R_{eq}^2}{\bar{a}_1} = \frac{0.392 \cdot 0.391^2}{0.0219} = 2.74 \text{ h} .$$

The discrepancy between the calculated and experimental (2.75 h) time of complete crystallization of the given ingot amounts to 0.4%, which can be considered to be quite satisfactory in both the accuracy and simplicity of the calculations. If the heat content of the solid and liquid phases is disregarded, the duration of solidification will be equal to 1.86 h, which is 32% smaller than the actual duration.

To calculate the surface temperature of the ingot at the instant of its complete solidification, from the graphs presented in Fig. 4 we find

$$\omega_1 \text{ Ko} = 0.07 ; \quad \omega_1 = 0.07/0.28 = 0.25 ; \quad \omega_2 \text{ Po} = 0 ; \quad \omega_2 = 0 ;$$

$$\Psi_{sur} \text{ Ko} = 0.063 ; \quad \Psi_{sur} = 0.063/0.28 = 0.225 ; \quad \Phi_{sur} = 0.345 .$$

Substitution of the data obtained into Eq. (3) yields

$$\theta_{sur}(l) = 0.345 + \frac{0.225}{1 + 0.25 + 0} = 0.525 ; \quad T_{sur} = T_m + (T_{sol} - T_m)\,\theta_{sur} = 30 + (1420 - 30)\,0.525 = 760\ ^{o}\text{C} .$$

Previously, in determining the mean values of the thermophysical characteristics of the metal, the average temperature was taken to be equal to 1195^0C. But if the average temperature of the ingot is calculated as:

$$\bar{T}_1 = (T_{sol} + T_{sur})/2 = (1420 + 760)/2 = 1090\ ^{o}\text{C}\ ,$$

then it is obvious that it differs from that taken previously by 9%. If necessary, the entire calculation can be repeated with a new value of $\bar{T}_1$.

However, ingots in the molds usually arrive at a stripper shop before the crystallization process completely terminates. And "stripping" of the ingots occurs to the instant of complete solidification as well.

In Fig. 5, the graph of the change in the value of the inverse velocity W^* of solidification of the cylindrical ingot is presented, from which it is obvious that beginning with $l \geq 0.8$ the solidification velocity increases rapidly. At this instant the decrease in the intensity of external heat removal (without a mold α is smaller than with a mold) virtually has little effect on the subsequent process of solidification. Moreover, the thickness $l \geq 0.8$ totally ensures the strength of the shell.

Similarly to the previous procedure, we determine the temperature of the ingot surface for l = 0.8 with the same initial data:

$$\omega_1\,\text{Ko} = 0.142\ ;\quad \omega_1 = 0.142/0.28 = 0.51\ ;\quad \omega_2\,\text{Po} = 0.14\ ;\quad \omega_2 = 0.14/9.4 = 0.015\ ;$$

$$\Psi_{sur}\,\text{Ko} = 0.052\ ;\quad \Psi_{sur} = 0.052/0.28 = 0.19\ ;\quad \Phi_{sur} = 0.47\ ;$$

$$\theta_{sur}\,(l) = 0.47 + \frac{0.19}{1 + 0.51 + 0.015} = 0.595\ ;$$

$$T_{sur} = T_m + (T_{sol} - T_m)\,\theta_{sur} = 30 + (1420 - 30)\cdot 0.595 = 857\ ^{o}\text{C}\ .$$

The time needed for the solidified layer of thickness l = 0.8 to be formed can be calculated from formula (4), whose components are partially presented in Fig. 5 in the following graphic form:

$$H^*/\text{Ko} = 0.84\ ;\quad H^* = 0.84\cdot 0.28 = 0.235\ ;$$

$$\beta_1\,\text{Ko} = 0.11\ ;\quad \beta_1 = 0.11/0.28 = 0.39\ ;$$

$$\beta_2\,\text{Po} = 0.425\ ;\quad \beta_2 = 0.425/9.4 = 0.045\ ;$$

$$\tau = 0.235\,(1 + 0.39 + 0.045) = 0.337\ ;$$

$$t = \frac{\tau R_{eq}^2}{a_1} = \frac{0.337\cdot 0.391^2}{0.0219} = 2.35\ \text{h}\ .$$

The simplicity of such calculations allows us to recommend the described above graphoanalytical method for practical engineering computations of the main criteria for the process of crystallization of metallurgical ingots.

NOTATION

T, temperature; ΔT_{mol}, temperature drop over the mold thickness; R, radius; t, time; τ, dimensionless time (Fourier number); $\bar{\tau}$, duration of complete solidification; τ_{tot}, duration of solidification with account for removal of superheating heat; τ^0, time of removal of superheating; τ^*, time of solidification of the ingot by Leibenzon's solution [5]; β_1 and β_2, correction functions that take into account the heat content of the solid and liquid phases (superheating) in calculation of the thickness of the solidified layer; θ, dimensionless tem-

perature; θ^*, dimensionless temperature obtained from Leibenzon's solution; $\xi = r/R$, dimensionless coordinate; r, running radius; x, running coordinate; $l = L/R$, thickness of the solidified layer; L, thickness of a growing solid skin; Ko, Kossovich criterion; ω_1 and ω_2, correction functions that take account into the heat content of the solid and liquid phases (superheating), respectively, in calculation of the temperature field; $K_a = a_2/a_1$, dimensionless parameter; a, thermal diffusivity, $K_T = (T_0 - T_{liq})/(T_{sol} - T_{in})$, dimensionless parameter; m, shape factor of the body ($m = 0$ for the plate; m = 1 for the cylinder; $m = 2$ for the sphere); α, heat transfer coefficient; Bi, Biot criterion; H^*, function that takes into account the law of movement of the crystallization front as a function of time (by Leibenzon's solution); Ψ_{sur}, dimensionless parameter; b, heated (cooled) portion of the ingot cross section. Subscripts: 0, initial; sol, solidus; liq, liquidus; m, medium; mol, mold; out, outer; in, inner; g, gap, rad, radiative; conv, convective; sur, surface; eq, equivalent; 1, solid phase; 2, liquid phase.

REFERENCES

1. L. S. Leibenzon, *Handbook of Oil-Field Mechanics*, Pt. 4, *Hydraulics* [in Russian], Moscow–Leningrad (1931).
2. S. M. Adams, *Liquid Metals and Their Solidification* [in Russian], Moscow (1962).
3 A. I. Veinik, *Theory of Solidification of a Casting* [in Russian], Moscow (1960).
4. B. Ya. Lyubov, *Theory of Bulk Crystallization* [in Russian], Moscow (1975).
5. L. S. Leibenzon, *Izv. Akad. Nauk SSSR, Ser. Geogr. Geofiz.*, No. 6, 1133–1175 (1939).
6. G. P. Ivantsov, in: *Heat Engineering of Ingots and Furnaces. Collection of Papers of the Central Scientific-Research Institute of Ferrous Metallurgy* [in Russian], Moscow (1960), pp. 60–104.
7. I. D. Semikin and E. M. Gol'dfarb, *Litein. Proizv.*, No. 2, 16–27 (1956).
8. Yu. A. Samoilovich, *Inzh.-Fiz. Zh.*, **11**, No. 5, 651–657 (1966).
9. Yu. S. Postol'nik, *Izv. Vyssh. Uchebn. Zaved., Mashinostroenie*, No. 1, 130–135 (1973).
10. Yu. S. Postol'nik, *Approximate Methods of Investigations in Thermal Mechanics* [in Russian], Kiev–Donetsk (1984).
11. Yu. S. Postol'nik, V. I. Timoshpol'skii, I. A. Trusova, and O. V. Dubina, *Lit'e Metallurgiya*, No. 1, 53–55 (2000).